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by G. Louis Smith

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SUMMARY

One of the sources of torques on near-earth satellites is the interaction of the earth's magnetic field with eddy currents induced in the electrically conducting parts of a spinning satellite. Initially, it is assumed that the geomagnetic field is that of a space-fixed dipole. The equation for the average torque, due to this effect, acting on a satellite of known properties is derived and presented. This equation shows the variation of the torque with the orbital parameters. The time history of the spin vector as influenced by this torque has been investigated. For the general case, the spin vector can be resolved into three orthogonal components which are damped exponentially at three different rates.

The analysis is extended to include the effect of the tilt of the geomagnetic dipole axis with respect to the earth's spin axis. The analysis and equations which include this effect closely parallel the preceding analysis and equations.

The generalized parameters, which can be used in analyzing the motion due to eddy currents of any spinning satellite in a near-circular orbit, are presented in graphical form.

Author

INTRODUCTION

One of the sources of torques on near-earth satellites is the interaction of the earth's magnetic field with eddy currents induced in the electrically conducting parts of a spinning satellite. This effect is considered in references 1 to 6. The problem consists of two parts: first, the calculation or measurement of the eddy-current torques on the satellite due to the presence of a given magnetic field, and second, the calculation of the torques and their effect on the angular motions of the satellite while in orbit around the earth. Studies of the first part of the problem are presented in the literature, but the latter part of the problem is treated only by estimates or else for very special cases. Sanduleak (ref. 7) sets up the equations for the spin-damping torque, in which the earth's magnetic field and orientation of the spin vector are considered as variables. Sanduleak programed these equations on an electronic data processing system for the torque computation, and numerical integration of the equations of motion yields the spin-rate history. However, only the damping component of torque is considered. It is shown by Vinti in reference 4 that there is also a torque component tending to precess the spin vector.

The purpose of this paper is to derive and present expressions for the average torques due to induced eddy currents acting on a satellite and to study the resulting time history of the spin motion. Examples of application of this study are the improved estimation of torques acting on a spinning satellite which must be oriented or for which the spin must be maintained within limits, and the calculation of the damping of satellite spin for comparison with observed variations.

SYMBOLS

$\left[A(i)\right]$	square matrix, defined by equation (14)
$\left[A'\right]$	square matrix, defined in equation (4)
a_i	coefficient of eigenvector where $i = 1,2,3$
$\left[B\left(i_I\right)\right]$	square matrix, defined by equation (34)
$\left[B\left(i_{I,\mu}\right)\right]$	square matrix, defined by equation (32)
b_{mn}	elements of $\left[B\left(i_I\right)\right]$ where $m,n = 1,2,3$
$\det \lvert\,\rvert$	determinant of matrix
$\vec{e}_i$	eigenvector where $i = 1,2,3$
$\vec{H}$	magnetic field vector
H_i	component of magnetic field vector where $i = x$, y, and z
$\left[I\right]$	identity matrix
I_{max}	maximum moment of inertia of satellite
i	orbital inclination
$\vec{i},\vec{j},\vec{k}$	unit vectors

K	interaction constant (see eq. (1))
k^2	earth's gravitation constant
$\vec{M}$	torque vector
P	period of satellite orbit
p	semilatus rectum of orbit
R_e	radius of earth
r	geocentric radius
s	geomagnetic dipole strength
$[T]$	transformation matrix (with subscript)
t	time
$\vec{X}$	coordinate vector
X,Y,Z	Cartesian coordinates
x,y,z	components along X, Y, Z axes
$\alpha,\beta,\gamma,\delta$	elements of $[A]$
ϵ	eccentricity
ζ	tilt angle of dipole
θ	true anomaly
η	argument of perigee
Λ_n	eigenvalue of $\left[B\left(i_I\right)\right]$ where $n = 1, 2, 3$
λ_n	eigenvalue of $[A]$ where $n = 1, 2, 3$
μ	angle of earth rotation
ν	angle from x_m to x_E
τ	time (nondimensional)
χ	angle from Y-axis to $\vec{e}_2$

ϕ satellite longitude

ψ satellite colatitude

$\vec{\omega}$ spin vector

ω spin rate

$[\ ]$ square matrix

$\{\ \}$ column matrix

Subscripts:

$(\ )_E$ earth-fixed reference

$(\ )_I$ space-fixed, or inertial, reference

$(\ )_M$ geomagnetic reference

$(\ )_o$ initial condition

r radial direction away from geocenter

ψ direction of increasing ψ

av average or mean value

Dots denote derivatives with respect to time

ANALYSIS

A Cartesian coordinate system is defined, with the Z-axis along the dipole axis and the X-axis passing through the ascending node of the intersection of the orbit plane with the geomagnetic equatorial plane. The Y-axis then completes the coordinate system, as shown in figure 1. The orbit orientation is described with respect to this system by the inclination to the geomagnetic equator i and the argument of the perigee η. The position of the satellite is given by the orbit angle θ or by the longitude ϕ and colatitude ψ within the coordinate system.

The torque due to eddy currents acting on a satellite is shown by Vinti (ref. 4) to be

$$\vec{M} = K\left(\vec{\omega} \times \vec{H}\right) \times \vec{H} \tag{1}$$

where K is a constant for the satellite spinning about a given body axis. (Small variations of K due to such effects as change of conductivity with satellite temperature are neglected in this paper.) Equation (1) is rewritten as

$$\vec{M} = K\left[\vec{H}\left(\vec{H} \cdot \vec{\omega}\right) - H^2\vec{\omega}\right] \tag{2}$$

The time required for the spin to decrease to one-half its original value due to eddy-current torques is in general on the order of a hundred days. (See, for example, refs. 1 to 3.) Hence, during a day, the change in spin rate is on the order of 1 percent, and during a single orbit, approximately 2 hours, the spin vector is very nearly constant. If the spin vector is assumed constant, the torque may be integrated with respect to time around one orbit, and an average torque is thus calculated:

$$\vec{M}_{av} = \frac{1}{P}\int_0^P \vec{M}\,dt = \frac{K}{P}\left[A'\right]\left\{\omega\right\} \tag{3}$$

where

$$\left[A'\right] = \int_0^P \begin{bmatrix} -\left(H_y^2 + H_z^2\right) & H_xH_y & H_xH_z \\ H_yH_x & -\left(H_z^2 + H_x^2\right) & H_yH_z \\ H_zH_x & H_zH_y & -\left(H_x^2 + H_y^2\right) \end{bmatrix} dt \tag{4}$$

It is next necessary to evaluate the elements of $\left[A'\right]$.

Evaluation of Matrix Elements

The geomagnetic field is assumed to be a dipole field and the tilt of the axis of the dipole with respect to the spin axis of the earth is neglected; therefore, the field is not a function of longitude and time. The magnetic field is thereby a function of the position of the satellite, which is given by the classical orbit relations.

The spherical components of a dipole field of strength p are

$$H_r = \frac{2s \cos \psi}{r^3} \tag{5a}$$

$$H_\psi = \frac{s \sin \psi}{r^3} \tag{5b}$$

or, in Cartesian coordinates:

$$\begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix} = \frac{s}{r^3} \begin{Bmatrix} 3 \sin \psi \cos \psi \cos \phi \\ 3 \sin \psi \cos \psi \sin \phi \\ 3 \cos^2 \psi - 1 \end{Bmatrix} \tag{6}$$

By orbit mechanics

$$\dot{\theta} = \frac{\sqrt{k^2 p}}{r^2} \tag{7}$$

and

$$r = \frac{p}{1 + \epsilon \cos \theta} \tag{8}$$

The angles ψ and ϕ are given by

$$\cos \psi = \sin i \sin(\theta + \eta) \tag{9}$$

and

$$\tan \phi = \cos i \tan(\theta + \eta) \tag{10}$$

The variable of integration in equation (3) is changed to θ; thus,

$$\vec{M}_{av} = \frac{1}{P} \int_0^P \vec{M}\, dt = \frac{1}{P} \oint \frac{\vec{M}}{\dot{\theta}}\, d\theta = \frac{1}{P\sqrt{k^2 p}} \oint \vec{M} r^2 d\theta \tag{11}$$

and

$$[A'] = \frac{1}{\sqrt{k^2 p}} \oint \begin{bmatrix} -\left(H_y^2 + H_z^2\right) & H_x H_y & H_x H_z \\ H_y H_x & -\left(H_z^2 + H_x^2\right) & H_y H_z \\ H_z H_x & H_z H_y & -\left(H_x^2 + H_y^2\right) \end{bmatrix} r^2 d\theta \tag{12}$$

Equations (8), (9), and (10) are used, after extensive manipulation, in equation (6) to express the components of $\vec{H}$ as functions of θ and the orbit parameters p, ϵ, i, and η. These expressions are substituted into equation (12) and the integration performed. It is found that terms involving ϵ and ϵ^3 vanish. If the analysis is restricted to near-circular orbits, the terms in ϵ^2 and ϵ^4 can be neglected, and $[A']$ is found to be

$$[A'] = -\frac{2\pi}{\sqrt{k^2 p}} \frac{s^2}{p^4} [A(i)] \tag{13}$$

where

$$[A(i)] = \begin{bmatrix} \alpha(i) & 0 & 0 \\ 0 & \beta(i) & \delta(i) \\ 0 & \delta(i) & \gamma(i) \end{bmatrix}$$

$$= \begin{bmatrix} 1\frac{3}{8} - \frac{3}{8}\cos^2 i & 0 & 0 \\ 0 & 2\frac{1}{2} - 4\frac{7}{8}\cos^2 i + 3\frac{3}{8}\cos^4 i & 3\frac{3}{8}\sin^3 i \cos i \\ 0 & 3\frac{3}{8}\sin^3 i \cos i & 1\frac{1}{8} + 2\frac{1}{4}\cos^2 i - 3\frac{3}{8}\cos^4 i \end{bmatrix} \tag{14}$$

Figure 2 shows the elements of $[A(i)]$ as a function of inclination i.

The orbital period is replaced by $\frac{2\pi p^{3/2}}{K\left(1 - \epsilon^2\right)^{3/2}}$, and the ϵ^2 term is neglected; thus, equation (3) for the average torque due to eddy currents becomes

$$\left\{M_{av}\right\} = -\frac{Ks^2}{p^6}\left[A(i)\right]\left\{\omega\right\} \tag{15}$$

This torque can be added to other torques, for example, those due to gravitation gradients, in order to study the motion of a satellite. In the present paper, however, the only external torque considered is that due to eddy currents.

It is well-known that for spinning satellites, because of internal dissipation of kinetic energy of rotation, the only stable axis of rotation is the axis of maximum momentum. After the transition to this spin motion is complete, the angular momentum of the satellite is simply $I_{max}\left\{\omega\right\}$, and the equation of the spin motion of the satellite becomes

$$I_{max}\left\{\dot{\omega}\right\} = -\frac{Ks^2}{p^6}\left[A(i)\right]\left\{\omega\right\} \tag{16}$$

The solution form

$$\left\{\omega\right\} = \left\{\omega\right\}_o e^{-\frac{Ks^2}{I_{max}p^6}\lambda t}$$

is substituted into equation (16), from which

$$\left[A(i)\right]\left\{\omega\right\}_o = \lambda\left\{\omega\right\}_o$$

or

$$\left(\left[A(i)\right] - \lambda\left[I\right]\right)\left\{\omega\right\}_o = 0 \tag{17}$$

which is simply a matrix eigenvalue problem. Equation (17) has nontrivial solutions only if

$$\det \left| \left[A(i)\right] - \lambda\left[I\right] \right| = \begin{vmatrix} \alpha - \lambda & 0 & 0 \\ 0 & \beta - \lambda & \delta \\ 0 & \delta & \gamma - \lambda \end{vmatrix} = 0 \tag{18}$$

It is noted that the x-component is uncoupled from the y- and z-components, that is, all terms in the first column and first row vanish, except $\alpha - \lambda$, and the problem is simplified considerably. The three solutions of equation (18) are

$$\left.\begin{aligned} \lambda_1 &= \alpha \\ \lambda_{2,3} &= \frac{\beta + \gamma}{2} \mp \sqrt{\left(\frac{\beta + \gamma}{2}\right)^2 + \left(\delta^2 - \beta\gamma\right)} \\ &= \frac{\beta + \gamma}{2} \mp \sqrt{\left(\frac{\beta - \gamma}{2}\right)^2 + \delta^2} \end{aligned}\right\} \tag{19}$$

The eigenvalues are plotted in figure 3 as a function of inclination.

The components of the corresponding unit eigenvectors are the minors of the determinant $\left| \left[A(i)\right] - \lambda\left[I\right] \right|$ obtained by moving across a row and normalizing and are

$$\left.\begin{aligned} \vec{e}_1 &= \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \\ \vec{e}_2 &= \left[\left(\gamma - \lambda_2\right)^2 + \delta^2\right]^{-1/2} \begin{Bmatrix} 0 \\ \lambda_2 - \gamma \\ \delta \end{Bmatrix} \\ \vec{e}_3 &= \left[\left(\beta - \lambda_3\right)^2 + \delta^2\right]^{-1/2} \begin{Bmatrix} 0 \\ -\delta \\ \beta - \lambda_3 \end{Bmatrix} \end{aligned}\right\} \tag{20}$$

The matrix $\left([A(i)] - \lambda[I]\right)$ is symmetric; hence the eigenvectors are orthogonal and the calculations are simplified considerably. Since $\vec{e}_1$ coincides with the X-axis, $\vec{e}_2$ and $\vec{e}_3$ are obtained from $\vec{j}$ and $\vec{k}$ by rotating them through some angle X as shown in figure 4. Thus

$$\left.\begin{aligned} \vec{e}_2 &= \begin{Bmatrix} 0 \\ \cos X \\ \sin X \end{Bmatrix} \\ \vec{e}_3 &= \begin{Bmatrix} 0 \\ -\sin X \\ \cos X \end{Bmatrix} \end{aligned}\right\} \tag{21}$$

Comparison of equations (20) and (21) shows that

$$\tan X = \frac{\delta}{\lambda_2 - \gamma} \tag{22}$$

This angle of rotation X is shown as a function of orbital inclination in figure 5. Finally, the spin vector may be written as

$$\vec{\omega} = a_1\vec{e}_1 e^{-\lambda_1\tau} + a_2\vec{e}_2 e^{-\lambda_2\tau} + a_3\vec{e}_3 e^{-\lambda_3\tau} \tag{23}$$

where

$$\tau = \left(\frac{K}{I_{max}}\frac{s^2}{p^6}\right)t \tag{24}$$

The a_i's are the components of $\vec{\omega}_0$ expressed in the $\vec{e}_i$ system and may be written as

$$\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos X & \sin X \\ 0 & -\sin X & \cos X \end{bmatrix} \begin{Bmatrix} \omega_{x,o} \\ \omega_{y,o} \\ \omega_{z,o} \end{Bmatrix} \tag{25}$$

where $\omega_{x,o}$, $\omega_{y,o}$, and $\omega_{z,o}$ are the components of $\vec{\omega}_o$ in the Cartesian coordinate system.

Thus far, it has been assumed, for simplicity, that the geomagnetic field is a dipole with the axis along the earth's spin axis. Consideration is now given to the effects of the tilt of the dipole axis with respect to the earth's spin axis.

Rotating Earth With Tilted Dipole Field

The geomagnetic field of the earth is approximated by a dipole field, the dipole axis being tilted 17° with respect to the earth's spin axis. (See ref. 7.)

In order to include the effect of the tilt of the magnetic dipole field with respect to the earth's spin axis, it is necessary to define three coordinate systems. (See fig. 6.) The space-fixed or inertial system X_I,Y_I,Z_I is oriented with the Z_I-axis along the earth's spin axis, and the X_I-axis along the intersection of the satellite orbit plane and the equator so that the X_I-axis passes through the ascending node. The earth-fixed system X_E,Y_E,Z_E is oriented with the Z_E-axis along the earth's spin axis and the X_E-axis passing through the intersection of the equator with the geomagnetic equator. Finally, the geomagnetic system X_M,Y_M,Z_M is defined with the Z_M-axis along the axis of the geomagnetic dipole and the X_M-axis passing through the ascending node of the orbit referenced to the geomagnetic equator. In addition, several angles are defined: i_I denotes the orbital inclination with respect to the earth's equator and i_M the orbital inclination with respect to the geomagnetic equator, μ is the angle between the X_I- and X_E-axes, ν the angle between the X_M- and X_E-axes, and finally ζ is the tilt of the dipole field with respect to the earth's spin axis, that is, the angle between Z_E and Z_M. It is seen in figure 6 that these angles are all related, in the spherical triangle ABC. The purpose of each coordinate system is as follows:

The inertial coordinate system provides a necessary fixed reference, as the other coordinate systems rotate with the earth. The analysis of the previous section was referenced to the geomagnetic coordinate system. The earth-fixed system serves as an intermediate coordinate system by which Euler angle rotations are made in transforming from the inertial to the geomagnetic coordinate system, and is further necessitated by the requirement that $d\mu/dt$, which is the spin rate of earth, is constant.

As is seen from figure 6, the X_E,Y_E,Z_E system is obtained by rotating the X_I,Y_I,Z_I system about the Z-axis through the angle μ, or

$$\vec{X}_E = \begin{Bmatrix} x_E \\ y_E \\ z_E \end{Bmatrix} = \begin{bmatrix} \cos\mu & \sin\mu & 0 \\ -\sin\mu & \cos\mu & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_I \\ y_I \\ z_I \end{Bmatrix} = \left[T_\mu\right]\vec{X}_I \tag{26}$$

The X_M,Y_M,Z_M system is next obtained by rotating the X_E,Y_E,Z_E system, first about the X_E-axis by the tilt angle ζ, and then about the Z_M-axis by the angle ν:

$$\vec{X}_M = \begin{Bmatrix} x_M \\ y_M \\ z_M \end{Bmatrix} = \begin{bmatrix} \cos\nu & -\sin\nu & 0 \\ \sin\nu & \cos\nu & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\zeta & \sin\zeta \\ 0 & -\sin\zeta & \cos\zeta \end{bmatrix} \begin{Bmatrix} x_E \\ y_E \\ z_E \end{Bmatrix} = \left[T_\nu\right]\left[T_\zeta\right]\vec{X}_E \tag{27}$$

Equations (26) and (27) yield

$$\vec{X}_M = \left[T_\nu\right]\left[T_\zeta\right]\left[T_\mu\right]\vec{X}_I \tag{28}$$

The components of the vector in the inertial coordinate system are given in terms of the components in the geomagnetic coordinate system by simply reversing the rotations and their order:

$$\vec{X}_I = \left[T_{-\mu}\right]\left[T_{-\zeta}\right]\left[T_{-\nu}\right]\vec{X}_M \tag{29}$$

Equation (15) was derived on the assumption of a dipole field constant in time. As the earth rotates, the geomagnetic axes also move and therefore equation (15) does not apply. If the variation of the dipole field is slow, equation (15) may be used as an approximation to the torque. During a single orbital period of the satellite, the change in the geomagnetic field at a point is considered to be small.

The following approximations are now made:

(1) Equation (15) is applicable

(2) The summation of angular impulses over 1 day may be approximated by integration

(3) The satellite spin vector does not change significantly in 1 day

In order to use equation (15), it is necessary only to reference the vectors to the geomagnetic coordinate system:

$$\left[T_\nu\right]\left[T_\zeta\right]\left[T_\mu\right]\left\{\vec{M}_{av}\right\}_I = -\frac{Ks^2}{p^6}\left[A\left(i_M\right)\right]\left[T_\nu\right]\left[T_\zeta\right]\left[T_\mu\right]\left\{\omega\right\}_I \tag{30}$$

from which

$$\left\{M_{av}\right\}_I = -\frac{Ks^2}{p^6}\left[B\left(i_{I,\mu}\right)\right]\left\{\omega\right\}_I \tag{31}$$

where $\left[B\left(i_{I,\mu}\right)\right]$ is given by

$$\left[B\left(i_{I,\mu}\right)\right] = \left[T_{-\mu}\right]\left[T_{-\zeta}\right]\left[T_{-\nu}\right]\left[A\left(i_M\right)\right]\left[T_\nu\right]\left[T_\zeta\right]\left[T_\mu\right] \tag{32}$$

and is seen to be simply the similarity transform of $\left[A\left(i_M\right)\right]$ from the geomagnetic coordinate system to the inertial coordinate system. The average daily torque is thus written as

$$\left\{M_{av}\right\}_I = -\frac{Ks^2}{p^6}\,\frac{1}{2\pi}\oint\left[B\left(i_{I,\mu}\right)\right]\left\{\omega\right\}_I\,d\mu = -\frac{Ks^2}{p^6}\left[B\left(i_I\right)\right]\left\{\omega\right\}_I \tag{33}$$

The matrix $\left[B\left(i_I\right)\right]$ which now takes the place of $\left[A\left(i_M\right)\right]$ is

$$\left[B\left(i_I\right)\right] = \frac{1}{2\pi}\int\left[B\left(i_{I,\mu}\right)\right]d\mu \tag{34}$$

The integration indicated in equation (34) was accomplished numerically. The matrix $\left[B\left(i_{I,\mu}\right)\right]$ being a similarity transform of $\left[A\left(i_M\right)\right]$, which is symmetric, is also symmetric; hence its integral, $\left[B\left(i_I\right)\right]$, is symmetric. The b_{12} and b_{13} (hence b_{21} and b_{31}, by symmetry) terms are found to be only a fraction of a percent of the larger terms in the matrix, and may be considered to be negligible. The remaining predominant matrix elements b_{11}, b_{22}, b_{33}, and b_{23} $\left(=b_{32}\right)$ are plotted as functions of i_I, the inclination of the orbit plane to the equator, in figure 7.

Equation (33) can be used to calculate the daily average torque due to eddy currents on a spinning satellite. The time history of the effect of this torque can be determined by the same analysis as used in the previous section; equations (15) to (25) are immediately applicable when the elements of $\left[B\left(i_I\right)\right]$ are substituted for the elements of $\left[A\left(i_M\right)\right]$. The eigenvalues of $\left[B\left(i_I\right)\right]$, Λ_1, Λ_2, and Λ_3 are shown in figure 8 as a function of i_I, and the angle χ_I between the eigenvectors of $\left[B\left(i_I\right)\right]$ and the space-fixed coordinate axes is shown in figure 9 also as a function of i_I.

Numerical Example

In order to illustrate the application of the theory developed in this paper, the spin decay of a typical satellite is calculated. For the calculations, the following numbers are used:

$$\text{Argument of perigee} = 142.3^\circ$$

$$I = 2.0 \times 10^7 \text{ gm-cm}^2$$

$$K = 50 \text{ dyne-cm-sec/gauss}^2$$

$$p = 1.15R_e$$

$$i_I = 52^\circ$$

and

$$s = 0.3131$$

which is normalized with respect to R_e^3. The spin vector is considered to be tangent initially to the orbit at the perigee, but in opposite direction from the velocity vector.

The initial spin vector, expressed in the space-fixed-coordinate system, is thus

$$\vec{\omega}_0 = \omega_0 \begin{Bmatrix} \sin 142.3^\circ & \\ -\cos 142.3^\circ & \cos 52^\circ \\ -\cos 142.3^\circ & \sin 52^\circ \end{Bmatrix} = \omega_0 \begin{Bmatrix} 0.612 \\ 0.487 \\ 0.624 \end{Bmatrix}$$

It is seen from figure 9 that for $i_I = 52^\circ$, $\chi_I = 46^\circ$; thus, by equation (25)

$$\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 46^\circ & \sin 46^\circ \\ 0 & -\sin 46^\circ & \cos 46^\circ \end{bmatrix} \vec{\omega}_0 = \omega_0 \begin{Bmatrix} 0.612 \\ 0.788 \\ 0.083 \end{Bmatrix}$$

From equation (24),

$$\tau = \frac{50}{2.0 \times 10^7} \frac{(0.313)^2}{(1.15)^6} t = 1.06 \times 10^{-7} t \text{ seconds} = 0.916 \times 10^{-2} t \text{ days}$$

The eigenvalues for $i_I = 52^\circ$ are shown by figure 8 to be

$$\lambda_1 = 1.26$$

$$\lambda_2 = 1.54$$

$$\lambda_3 = 1.14$$

By using equation (21) for the eigenvectors, the equation for the spin vector (eq. (23)) thus becomes

$$\frac{\vec{\omega}}{\omega_1} = 0.612\begin{Bmatrix}1\\0\\0\end{Bmatrix}e^{-0.0115t} + 0.788\begin{Bmatrix}0\\0.695\\0.719\end{Bmatrix}e^{-0.0141t} + 0.083\begin{Bmatrix}0\\-0.719\\0.695\end{Bmatrix}e^{-0.0104t}$$

$$= \begin{Bmatrix}0.612\\0\\0\end{Bmatrix}e^{-0.0115t} + \begin{Bmatrix}0\\0.548\\0.567\end{Bmatrix}e^{-0.0141t} + \begin{Bmatrix}0\\-0.060\\0.058\end{Bmatrix}e^{-0.0104t}$$

CONCLUDING REMARKS

An analysis has been made to determine the average, or net, torques due to magnetically induced eddy currents, which act on a spinning near-earth satellite. Initially, the earth's magnetic field is assumed to be that of a space-fixed dipole. An equation for the torque vector, which is a linear function of the satellite spin vector, is developed, including the orbital parameters. It is found that the first- and third-order terms in eccentricity vanish identically; second- and fourth-order terms are neglected. The effect of orbital inclination is given by a matrix function, since in general the torque vector is not parallel to the spin vector.

The time history of the spin vector subject to this torque has been investigated. It is found that, for the general case, the spin vector can be resolved into three orthogonal components which are damped exponentially at three different rates.

The effect of the tilt of the geomagnetic dipole axis with respect to the earth's spin axis has been considered for both the torque and the resulting spin history. The analysis and resulting equations closely parallel those of the foregoing analysis which did not consider this effect. The two analyses, with and without consideration of the dipole tilt, yield qualitatively similar results, although they differ slightly quantitatively, as is to be expected.

The generalized parameters which can be used in analyzing the motion, due to eddy current torques for any spinning satellite in a near-circular orbit are presented in graphical form.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., December 10, 1963.

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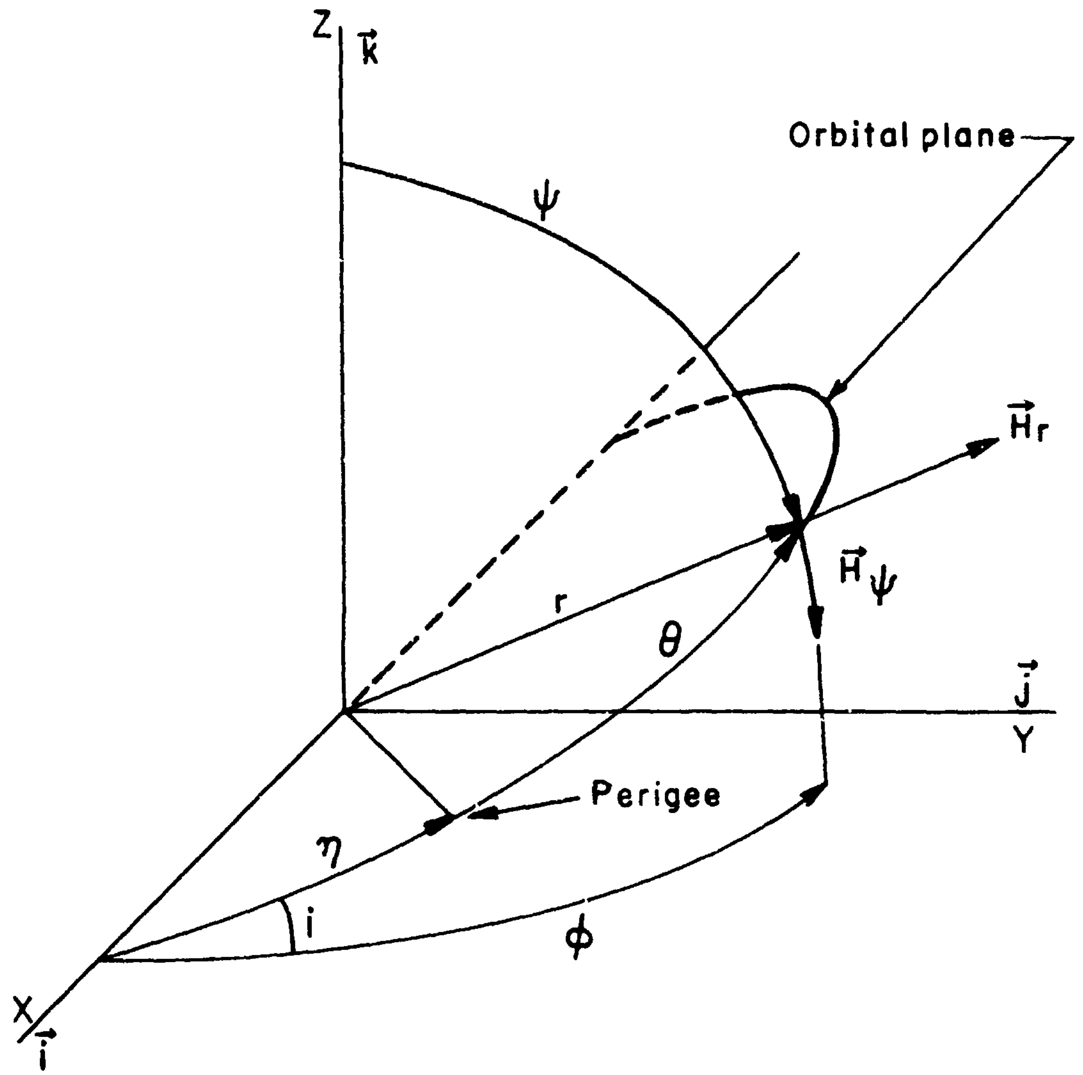

Figure 1.- Coordinate systems and symbols used for analysis of noninclined dipole. XY plane is earth equatorial plane, dipole axis is along Z-axis.

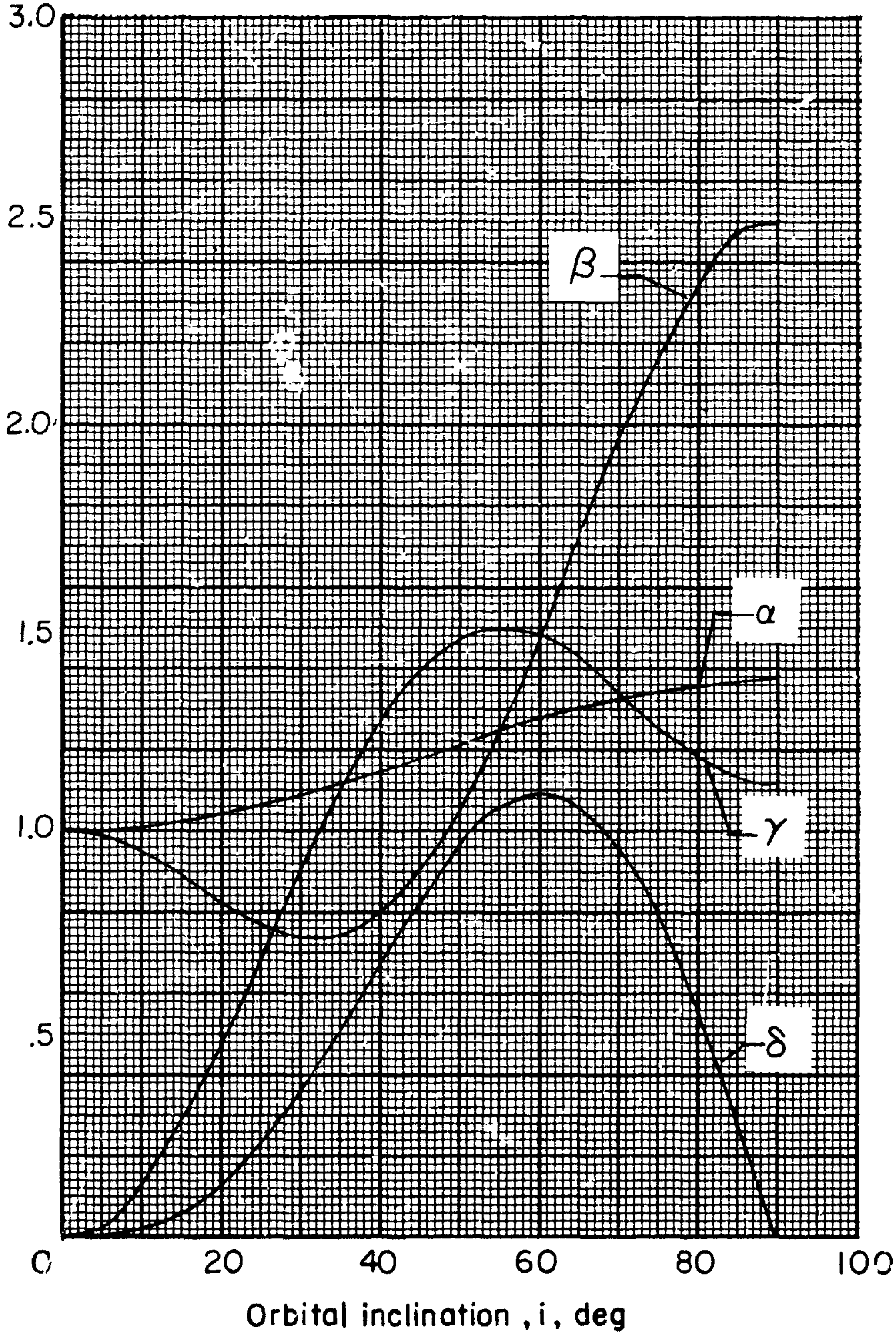

Figure 2.- Elements o. [A(i)] matrix as a function of orbital inclination.

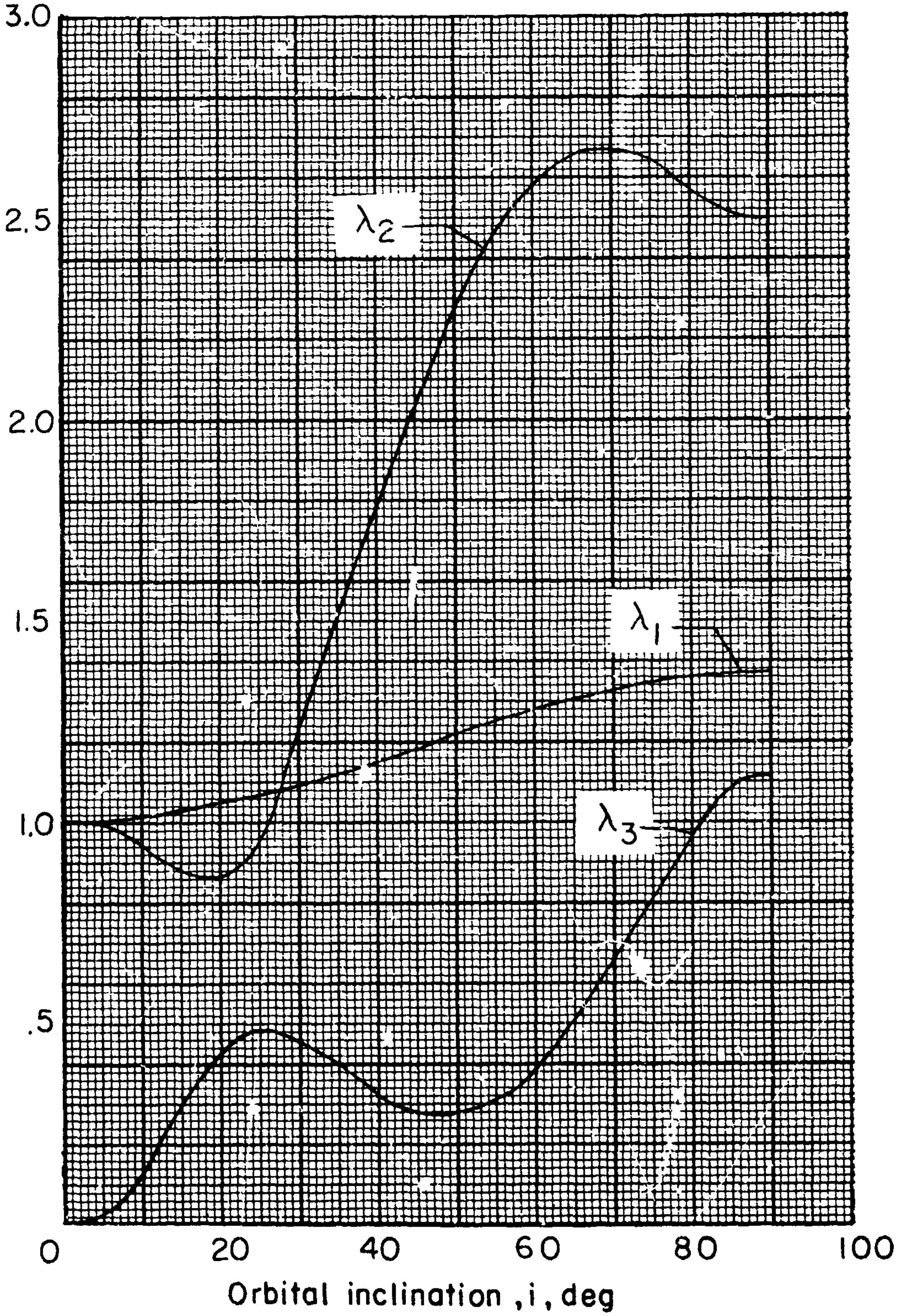

Figure 3.- Eigenvalues of [A(i)] matrix as a function of orbital inclination.

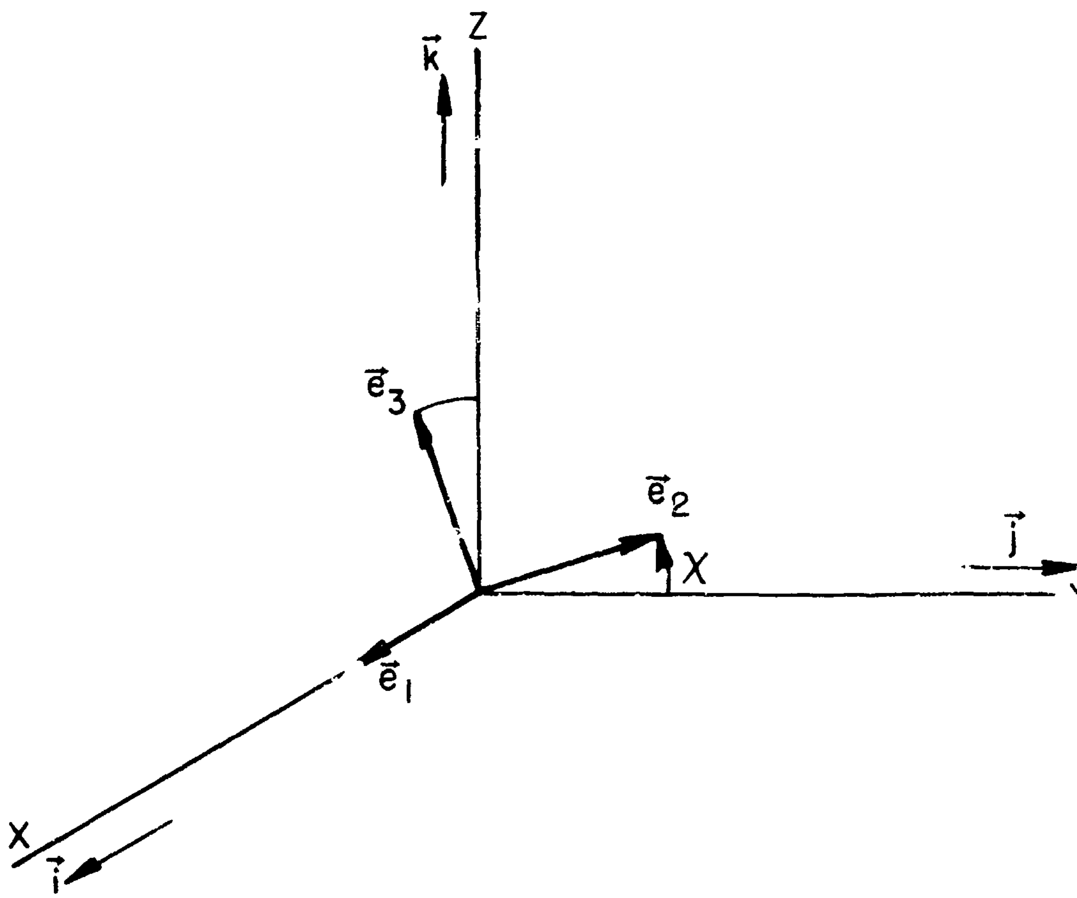

Figure 4.- Relation between coordinate systems and eigenvectors.

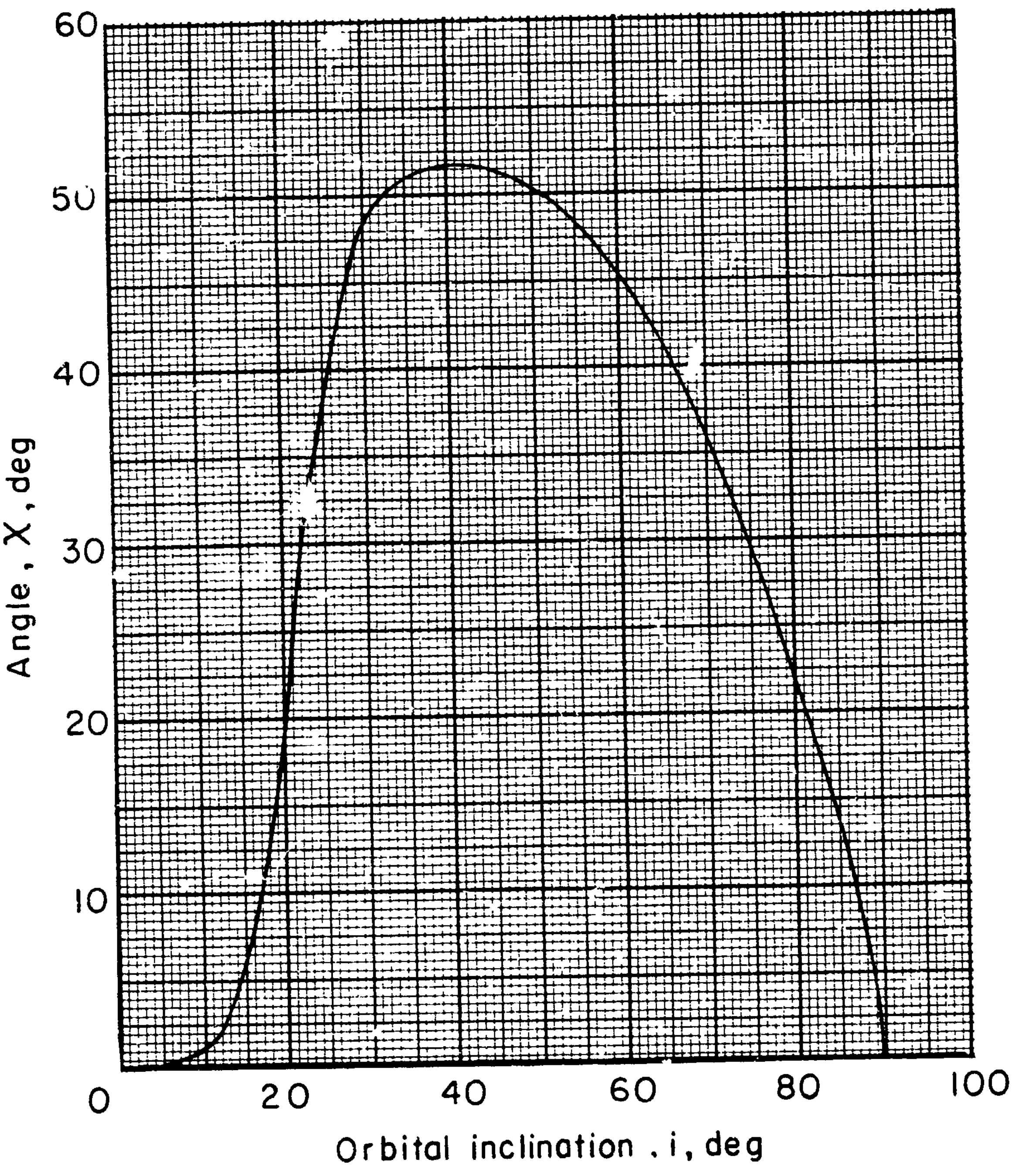

Figure 5.- Angle X between eigenvectors and coordinate axes as function of orbital inclination i.

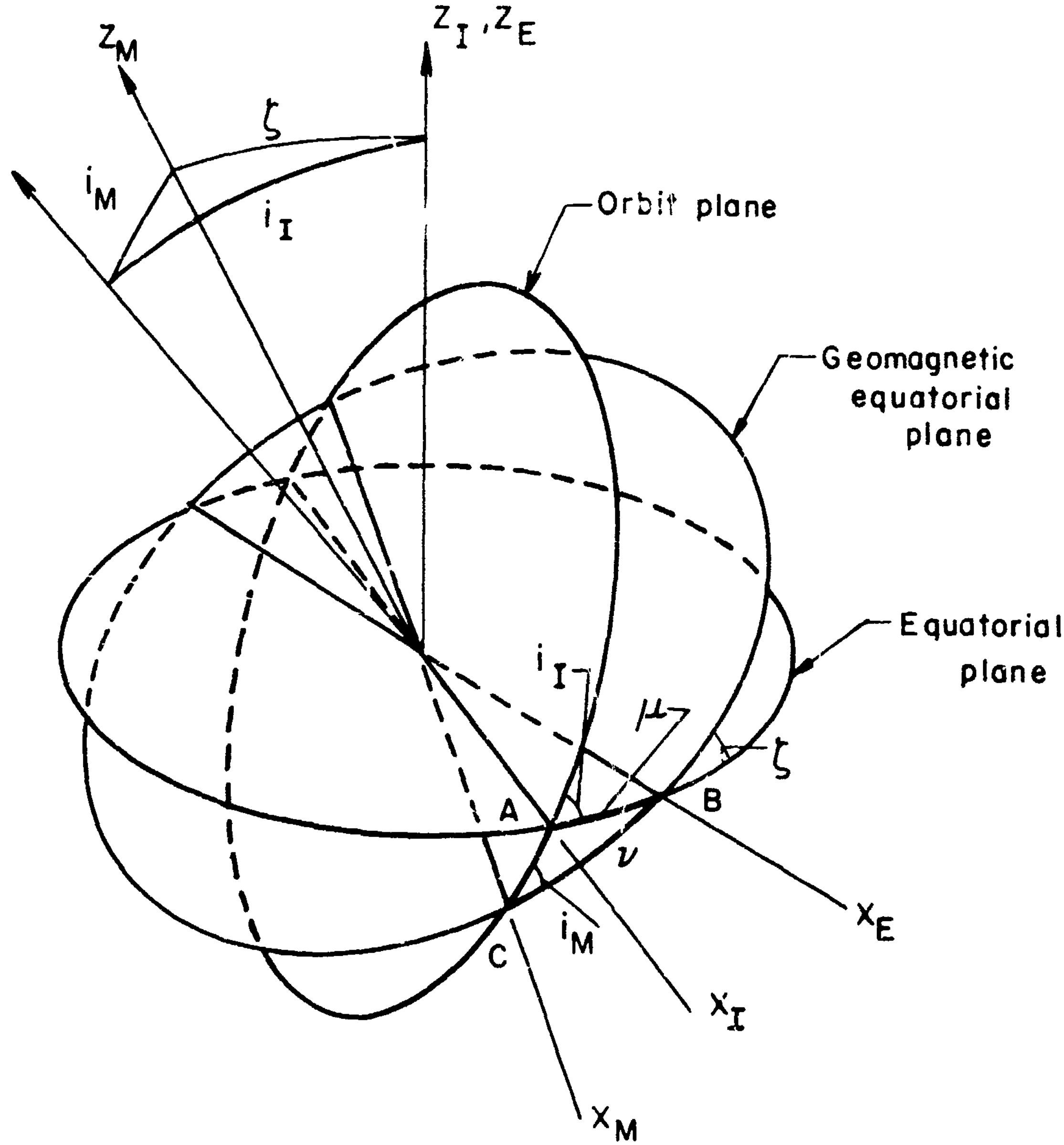

Figure 6.- Coordina : systems used for calculation of torques due to tilted dipole.

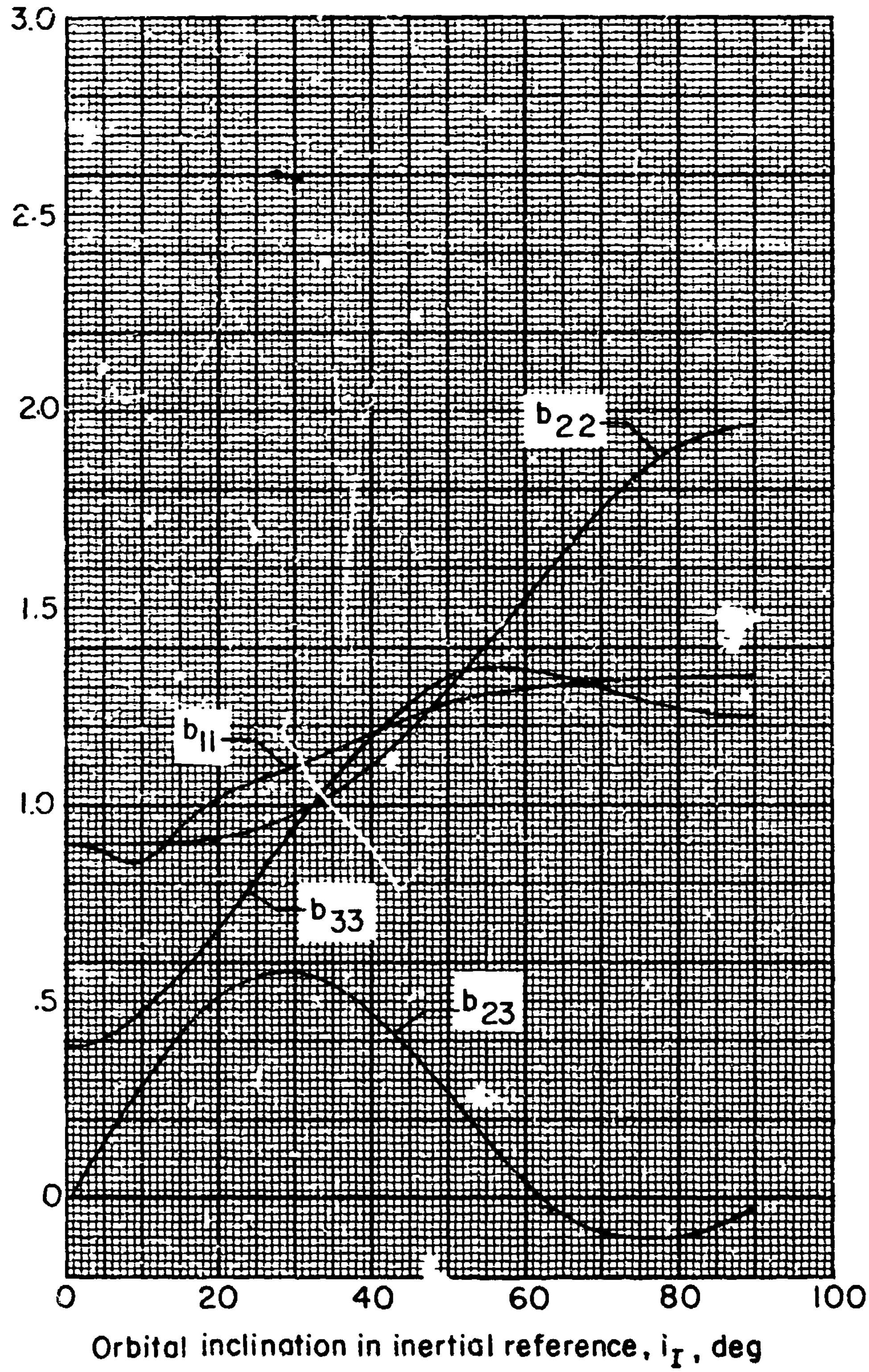

Figure 7.- Elements of $\left[B(i_I)\right]$ matrix as function i_I.

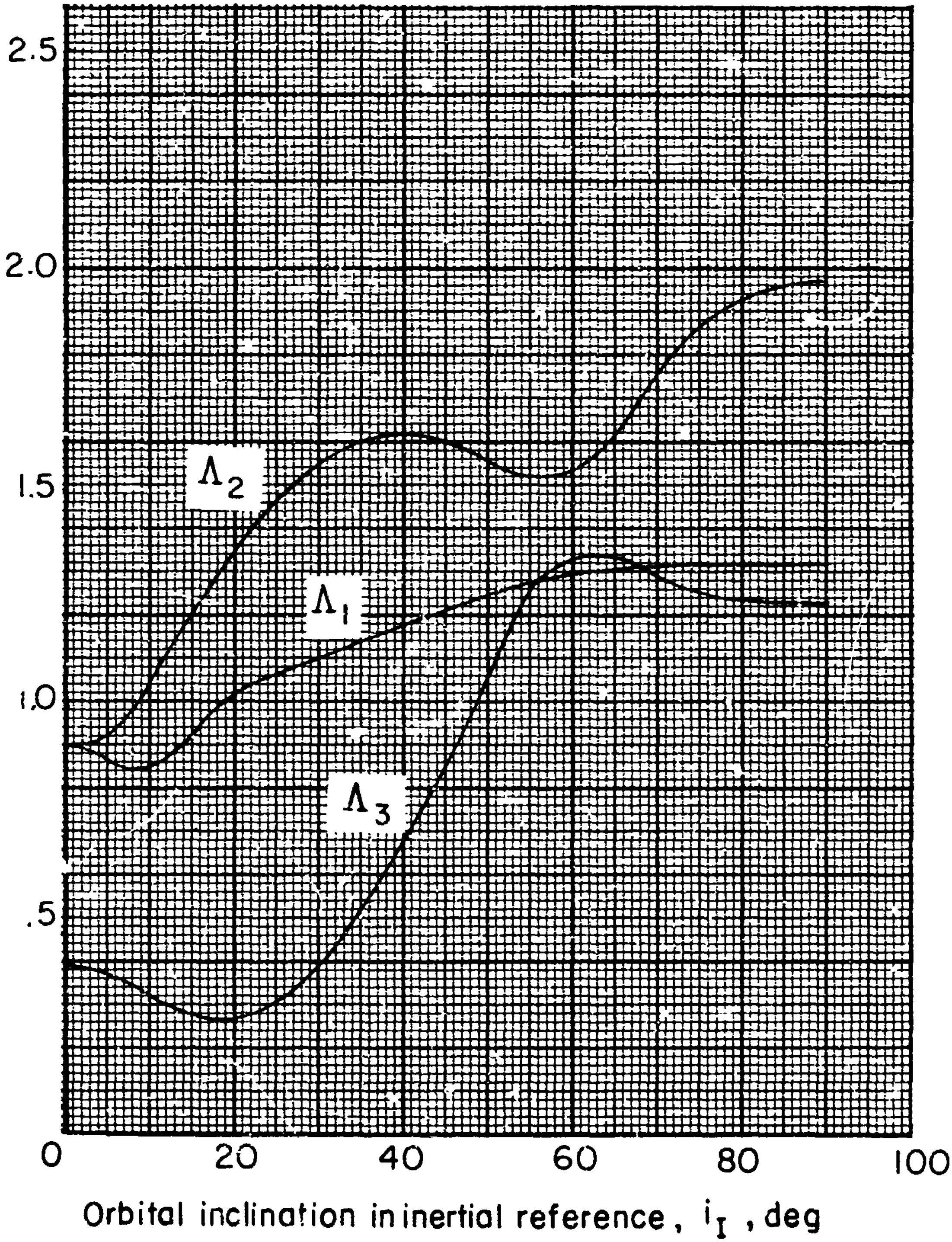

Figure 8.- Eigenvalues of $\left[B(i_I)\right]$ matrix as a function of orbital inclination.

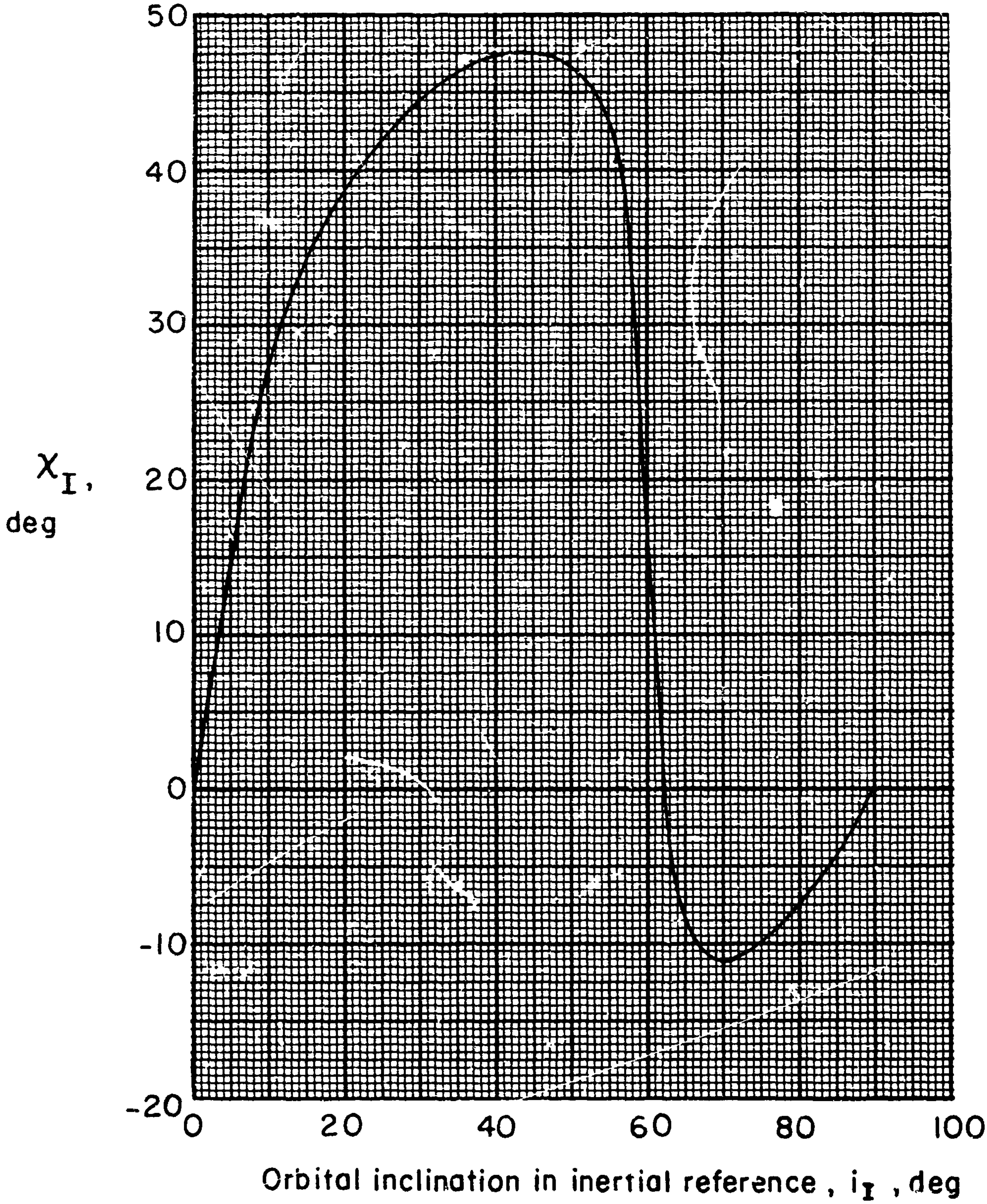

Figure 9.- Angle χ_I between eigenvectors and coordinate axes as function of orbital inclination in inertial reference i_I.

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